

# Genuine multipartite Einstein-Podolsky-Rosen steering

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We develop the concept of genuine  $N$ -partite Einstein-Podolsky-Rosen (EPR) steering. This nonlocality can be realized as a multiparty EPR paradox, and is a key resource for quantum secret sharing. Useful properties emerge that are not guaranteed for genuine multipartite entangled states. We derive EPR steering inequalities to demonstrate genuine multipartite EPR steering for GHZ, W and Gaussian continuous variable (CV) states.

Bell's seminal work showed that quantum mechanics is not equivalent to any local hidden variable theory (LHV) [1], but this work was a study of nonlocality between two particles only. Svetlichny asked whether quantum mechanics could exhibit a *genuine* three-body nonlocality [2], in which case the nonlocality cannot be simulated by any nonlocality that might exist between only two bodies. These ideas are crucial to understanding the full nature of the transition from the quantum to the classical regime [3–6]. Collins and co-workers [5] revealed that  $N$  party Greenberger-Horne-Zeilinger (GHZ) states can exhibit a genuine Bell nonlocality among  $N$  sites, and experiments have reported violation of Svetlichny inequalities using GHZ states [6]. The experimental violation however, is limited to  $N = 3$ , and to systems of only one qubit (photon) per site.

Our knowledge of multipartite entanglement on the other hand is much more established. Experimental signatures have been developed, for both continuous variable (CV) [7] and qubit systems [8, 9]. There has been experimental evidence in both cases [9–11], with the generation of fourteen entangled qubits in ion-traps [12] and recent reports of CV entanglement of up to eight light modes [13]. However, entanglement does not demonstrate nonlocality [14, 15], and it is widely appreciated that the detection of Bell nonlocality is far more challenging [16]. Observation of genuine  $N$ -partite Bell nonlocality is increasingly difficult for systems of very high dimension and for CV measurements [3]. Despite this, there is also increasing awareness that genuine nonlocality is not only fundamentally significant, but can be specifically required for certain quantum information tasks [15, 17, 18].

In this Letter, we investigate an intermediate type of genuine  $N$ -body nonlocality. As it is potentially less susceptible to noise and decoherence than genuine Bell violations, it is therefore more accessible to experiment. We consider genuine multipartite forms of Einstein-Podolsky-Rosen (EPR) steering. Steering has only recently been identified as a distinct type of nonlocality [15, 19], different to both entanglement and Bell's nonlocality, and is realised in experiments that reveal an EPR paradox [20, 21]. Work by Wiseman and co-workers [15] formalised Schrodinger's concept of "steering", that an observer can apparently instantaneously influence a

distant system, by making local measurements. Multipartite EPR steering has been studied for qubits [22] and qudits [23]. However, this work did not examine genuine nonlocality, in which the nonlocality is necessarily shared among all observers.

We show that it is possible to obtain genuine multipartite EPR steering in very different sorts of systems to those so far predicted for multipartite Bell nonlocality. To date, EPR steering has been verified in only a few experiments, including for very high efficiencies in CV Gaussian optical systems [21, 24, 25] and without detection loopholes for photons [26–28], but the focus has been on the bipartite case.

Here, we formalise the meaning of genuine multipartite EPR steering, and derive criteria to detect it. We show how to verify  $N$ -partite steering for GHZ and W states, as well as for CV Gaussian systems, and give efficiency bounds to do so conclusively. Our work therefore opens up possibilities to demonstrate an  $N$ -partite EPR nonlocality ( $N > 2$ ) unambiguously for qubit sites, whether by using photons [6, 26, 28] or ions [10, 12], and to test the existence of the strongest form of nonlocality so far predicted to distribute over many sites with systems in the continuous (CV) limit.

Further, we show that genuine multipartite EPR steering is **not** equivalent to genuine multipartite entanglement. We prove two properties that apply to multipartite EPR steering, that are useful to quantum secret sharing protocols [29], and are not guaranteed by multipartite entanglement.

*Genuine  $N$  partite nonlocality:* We consider  $N$  spatially separated systems at sites  $j = 1, \dots, N$ , and ask how to derive criteria for genuine  $N$ -party nonlocality, so that we can conclude nonlocality to be shared among *all*  $N$  parties.

The strongest form of nonlocality is Bell's nonlocality, in which all Local Hidden Variable (LHV) models are falsified [1]. Denoting the hidden variables that specify the predetermined nature of the system by the set  $\{\lambda\}$ , all LHV theories will imply the fully separable LHV model  $\langle \prod_{j=1}^N X_j \rangle = \int_{\lambda} d\lambda P(\lambda) \prod_{j=1}^N \langle X_j \rangle_{\lambda}$  for which  $j - 1$  factorisations in the integrand are apparent, due to the assumption of locality between all sites  $j$ . Here  $X_j$  are the possible results for a measurement  $\hat{X}_j$  at site  $j$ ,  $\langle X_j \rangle_{\lambda}$  is

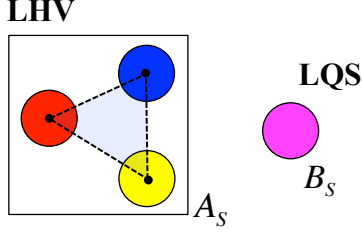


Figure 1. Depiction of the local-nonlocal hybrid model for  $N = 4$ , where three sites can share a Bell nonlocality, but four cannot. The group  $A_s$  “steers” system  $B_s$ , if this model fails, when it is also constrained that  $B_s$  be consistent with local quantum state statistics. Genuine 4-partite EPR steering is confirmed if any group of three can steer the remaining fourth system (Result 1).

the expected value of  $X_j$  for a given set  $\{\lambda\}$ , and  $P(\lambda)$  the hidden variable probability distribution function. Bell’s nonlocality is demonstrated when the LHV model fails.

Genuine multipartite Bell nonlocality can be tested if we construct a hybrid local-nonlocal model in which we allow Bell nonlocality to exist, but only if shared among  $N - 1$  or fewer parties [2, 5]. Thus, the fully separable LHV model becomes only partially separable, with separability retained between any two groups  $A$  and  $B$  of  $N - 1$  and 1 parties respectively. We label the possible ways of splitting the sites into two such groups by the index  $s$ . The Svetlichny hybrid local-nonlocal model is

$$\langle \prod_{j=1}^N X_j \rangle = \sum_s P_s \int_{\lambda} d\lambda P_s(\lambda) \langle \prod_{j \in A_s} X_j \rangle_{s,\lambda} \langle \prod_{j \in B_s} X_j \rangle_{s,\lambda} \quad (1)$$

for any  $\sum_s P_s = 1$ . Violation of all models (1) confirms the Bell nonlocality to be genuinely “ $N$ -partite” [2, 5].

We next consider the three different types of nonlocality – Bell nonlocality, steering, and entanglement – that may exist between two sites, as introduced by Wiseman and co-workers [15, 30]. Following those authors, the fully separable LHV model becomes a quantum separable model when there exists a local quantum density operator  $\rho_j^\lambda$  such that  $\langle X_j \rangle_\lambda = \text{Tr}(\rho_j^\lambda X_j)$ , for each  $j$ . In this case, the system is described by a fully separable density matrix  $\rho$  and failure of the model implies entanglement. To test for genuine  $N$ -partite entanglement, a partially separable model is used [7], where  $\rho = \sum_s P_s \int_{\lambda} d\lambda P_s(\lambda) \rho_{A_s}^\lambda \rho_{B_s}^\lambda d\lambda$  and  $\rho_{A_s}^\lambda$  is a density operator, not necessarily factorisable, for the sites  $A_s$  (similarly  $\rho_{B_s}^\lambda$ ). This is equivalent to the hybrid local-nonlocal model (1) but with the further constraint that moments for  $A_s$  and  $B_s$  each arise from a quantum density matrix,  $\rho_{A_s}^\lambda$  and  $\rho_{B_s}^\lambda$ , respectively. The failure of all such models demonstrates genuine  $N$ -partite entanglement.

Now we turn to the case of steering. Following Ref. [15], we impose the asymmetric constraint on the model (1) with  $P_s = 1$ , that there exists a quantum density op-

erator  $\rho_{B_s}^\lambda$  for the group of sites labelled  $B_s$ , but not for the group collectively labelled  $A_s$ . Failure of this model, called a local hidden state (LHS) model, demonstrates steering of system  $B_s$  by  $A_s$  [15]. Such steering can be confirmed through the violation of EPR-steering inequalities [15, 19, 30].

A hierarchy of nonlocality is implied by the definitions [15]. The local quantum state (LQS) description  $\rho_{A_s}^\lambda$  is a particular example of a LHV one. Hence, Bell nonlocality between  $A_s$  and  $B_s$  implies both steering and entanglement, and steering implies entanglement (but not Bell nonlocality). Unlike the other two nonlocalities, EPR steering is directional: that  $A$  can steer  $B$  does not imply that  $B$  can steer  $A$  [25].

A definition of genuine multi-partite steering now follows naturally. Genuine  $N$ -partite steering exists iff it can be shown that a steering nonlocality is necessarily shared among all  $N$  (or more) sites. This means that the system cannot be described by any state for which steering is confined to be shared among  $N - 1$  or fewer sites. In this paper, we say two parties “share steering” if (at least) one can be shown to steer the other. The meaning of  $N$  parties sharing steering is then defined recursively.

We next prove three results for multi-partite EPR steering. The first leads to simple criteria for detecting multipartite EPR steering; the second and third give properties that make multipartite EPR steering useful.

**Result 1:** (a) The failure of model (1) where each group  $B_s$  consists of one LQS site and group  $A_s$  consists of  $N - 1$  sites is sufficient to demonstrate genuine  $N$ -partite steering. (b) This amounts to confirming that any group of  $N - 1$  parties can “steer” the last party (Fig. 1).

*Proof:* (a) The model allows Bell nonlocality and hence, by the hierarchy of nonlocality, steering, to be shared among any  $N - 1$  sites. Genuine  $N$ -partite steering is thus demonstrated if the model is falsified. (b) If the model fails for each case where  $P_s = 1$ , so that each group  $A_s$  steers  $B_s$ , then it is straightforward to show [32] that the model must fail for all possible combinations of  $P_s$ .

**Result 2:** Demonstration of genuine  $N$ -partite steering via the method of Theorem 1 implies there exists a group  $A_s$  of  $N - 1$  sites that collectively steers the remaining site  $B_s$ . Collective steering means that the steering of site  $B$  cannot be demonstrated by measurements on  $N - 2$  or fewer sites alone.

*Proof:* The case of a system  $B_s$  being steered by a group of fewer than  $N - 1$  parties is encompassed in the assumption of steering nonlocality of up to  $N - 1$  parties, and cannot therefore demonstrate genuine  $N$ -party steering. Thus, in this special example, genuine  $N$ -party steering implies a collective steering of at least one single site  $B_s$  by the remaining  $N - 1$  sites.

**Result 3:** If it can be proved by violation of an EPR steering inequality involving two observables that group

A “steers”  $B$ , then there can be no third group  $C$  completely independent of  $A$  that can also violate the same inequality. For EPR inequalities using  $n$  observables, there can be no more than  $n - 2$  independent parties that can also steer  $B$ .

*Proof:* To illustrate the method of proof, consider the EPR steering inequality [31] based on the Heisenberg relation  $\Delta X_B \Delta P_B \geq 1$  for conjugate operators of  $B$ :  $A$  can steer  $B$ , if  $\Delta_{inf,A} X_B \Delta_{inf,A} P_B < 1$  where  $\Delta_{inf,A} X_B$  is the inference error in the prediction  $X_{pred}$  of  $X_B$  based only on measurements of system  $A$  [15, 30]. Now suppose  $C$  can also steer  $B$  by  $\Delta_{inf,C} X_B \Delta_{inf,C} P < 1$ . This creates a real contradiction with the uncertainty principle and is therefore impossible within the bounds of quantum mechanics [33]. More generally, the proofs follow, because  $A$  and  $C$  can perform measurements simultaneously, and the state of  $B$  conditioned on their joint outcomes is definable as a quantum state.

It is not immediately obvious whether the properties given by Results 2 and 3, which are useful for secure quantum communication, are guaranteed for multipartite entangled states. We show this *not* to be the case, but first we derive criteria to detect genuine  $N$  partite EPR steering.

*$N$ -partite GHZ qubit states:* Consider the  $N$ -spin GHZ state  $\frac{1}{\sqrt{2}}\{|\uparrow\rangle^{\otimes N} - |\downarrow\rangle^{\otimes N}\}$ . Here  $|\downarrow\rangle_j, |\uparrow\rangle_j$  are eigenstates of the Pauli spin  $\sigma_z^j$  of the  $j$ th particle. In this case, steering of any single party by the remaining group of  $N - 1$  parties can be demonstrated by way of the GHZ EPR paradox [15, 21, 30]. The GHZ state is an eigenstate of  $\sigma_x^N \prod_{j=1}^{N-1} \sigma_y^j$  (and all other products arising from the permutations among the  $N$  sites), with eigenvalue  $+1$  [34]. Thus, any group of  $N - 1$  observers is able to predict the outcome of the spin  $\sigma_x^N$  of the  $N$ th particle, by measuring  $\sigma_{x,pred}^N = \prod_{j=1}^{N-1} \sigma_y^j$ . The conditional variance  $\Delta_{inf}^2 \sigma_x^N = \langle (\sigma_x^N - \sigma_{x,pred}^N)^2 \rangle$  is zero for the GHZ state. In a similar way, the spin  $\sigma_y$  of the  $N$ th particle can be predicted if the  $N - 1$  observers measure the spin product  $\sigma_{y,pred}^N = \sigma_x^{N-1} \prod_{j=1}^{N-2} \sigma_y^j$ . The conditional variance  $\Delta_{inf}^2 \sigma_y^N = \langle (\sigma_y^N - \sigma_{y,pred}^N)^2 \rangle$  is also zero for the GHZ state. If the observers are spatially separated, the EPR argument [20] is that the  $N$ th particle cannot be changed by measurements made by the remaining  $N - 1$  observers and hence that the spin components  $\sigma_x^N$  and  $\sigma_y^N$  of the  $N$ th particle are both predetermined. Since quantum mechanically it is not possible to predetermine both spin components simultaneously, an EPR paradox is obtained [20]. The GHZ EPR paradox can be realised even where conditional variances are nonzero if [30, 31]

$$\Delta_{inf}^2 \sigma_x^N + \Delta_{inf}^2 \sigma_y^N < 1. \quad (2)$$

This follows, since for any quantum state that could be used to simulate the predetermined nature of the spins, the uncertainty relation  $\Delta^2 \sigma_x + \Delta^2 \sigma_y \geq 1$  holds [22]. Thus, observation of (2) implies an EPR paradox, and

hence an EPR steering of the  $N$ th spin system ( $B_s$ ) by the group ( $A_s$ )  $j = 1, \dots, N - 1$  [21, 30]. By Result 1, the certification of (2) for all  $N$  choices of the  $N$ th site will imply genuine  $N$ -partite EPR steering. The symmetry of the GHZ state tells us this will be satisfied. In this way,  $N$ -partite GHZ states can be shown to exhibit  $N$ -partite EPR steering.

The properties of Results 2 and 3 therefore hold for the GHZ state. Collective steering is apparent, since  $N - 1$  parties must collaborate, to generate the predictions  $\sigma_{x,pred}^N, \sigma_{y,pred}^N$ . The predictions are obtained by measurements on each of the other sites, as necessary for secret sharing protocols [29]. The EPR steering inequality (2) depends on two observables  $\sigma_x$  and  $\sigma_y$ . Hence Theorem 3 applies, to guarantee that the inference variances  $\Delta_{inf,C}^2 \sigma_x^N$  and  $\Delta_{inf,C}^2 \sigma_y^N$  for any third group of observers  $C$  cannot be low enough to satisfy (2). The result is universal, regardless of what measurements or devices are selected by  $C$ . If verified between two groups, the EPR steering inequality (2) thus gives a means to certify the security of the information shared.

A related consequence of Result 3 is that any two-observable inequality such as (2) cannot be realised when the detection efficiencies of the steering group are reduced to  $\eta \leq 0.5$  at each site. This follows, since 50/50 beam splitters at each site enable formation of two steering groups that can have identical inference statistics. EPR steering inequalities with  $n$  observables can be verified with lower losses,  $\eta > 1/n$  (by the same argument).

Multipartite EPR steering inequalities involving  $n$  observables can be obtained from those introduced in [19] and used in [26, 28] and [27] to achieve loophole-free EPR steering in the bipartite case. These inequalities are  $S_n = \frac{1}{n} \sum_{k=1}^n A_k \sigma_k^B > C_n(\eta_A)$  where  $\sigma_k^B \equiv \sigma_k^N$  is a spin observable for the  $N$ th system ( $B$ ),  $A_k$  is a quantity evaluated by measurements made by the steering group ( $A$ ), of all the remaining  $N - 1$  observer and  $\eta_A$  is the overall detection efficiency of group  $A$ . We find for any spin choice  $k$  that the value  $A_k$  measured in the bipartite case can be measured collectively by the group  $A$  in the  $N$ -partite GHZ case. The quantum prediction for  $S_n$  is the same as for a maximally entangled bipartite Bell state, provided  $\eta_A = \eta^{N-1}$ . The optimal  $C_n(\eta)$  and  $k$  are evaluated in [28], where it is shown that  $S_n > C_\eta(\eta_A)$  for any  $\eta_A > 1/n$ , making loophole-free photonic demonstration of  $N$ -partite EPR steering experimentally feasible.

*$W$  states:* Whether the  $N$ -partite  $W$  state [9] can show genuine EPR steering is not clear, a priori. The tripartite  $W$  state is written  $|W\rangle = \frac{1}{\sqrt{3}}\{|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle + |\downarrow\downarrow\uparrow\rangle\}$ , using standard notation. Our method, using inequality (2), establishes  $\Delta_{inf} \sigma_z^3 = 0$  and  $\Delta_{inf}^2 \sigma_x^3 = 1/6$  (based on measurements  $\sigma_x^1 \sigma_x^2$ ) and confirms genuine 3-party EPR steering for this state.

*CV GHZ states:* Consider  $N$  harmonic oscillators (fields) at sites  $j$ , with boson operators  $a_j$ . The quadra-

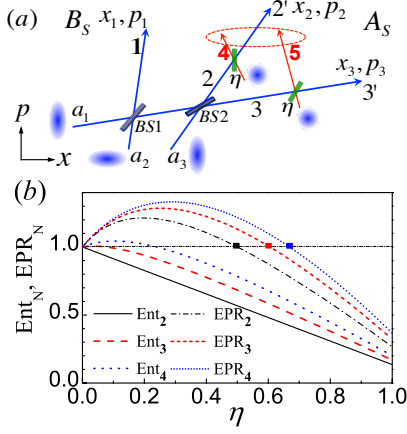


Figure 2. (a) Genuine CV tripartite entanglement for beams  $\{1, 2, 3\}$  can be generated via squeezed states and beam splitters (BS) [11]. (b) Genuine tripartite entanglement exists for  $\{1, 2', 3'\}$  when  $\text{Ent}_3 < 1$  whereas EPR steering of  $\{1\}$  by  $\{2', 3'\}$  is confirmed when  $\text{EPR}_3 < 1$ . Once  $\eta = 0.5$ , observers at beams  $\{4, 5\}$  and  $\{2, 3\}$  have identical inferences of the amplitudes of  $\{1\}$ , and the EPR steering is necessarily destroyed (Result 3). Cases  $N = 2, 4$  are also shown.

ture amplitudes  $x_j, p_j$  are given by  $a_j = x_j + ip_j$ . The Heisenberg relation is  $\Delta x_j \Delta p_j \geq 1$ , so violation of the associated EPR steering inequality gives the condition

$$\Delta_{\text{inf}} x_j \Delta_{\text{inf}} p_j < 1 \quad (3)$$

which if satisfied for each  $j$  confirms  $N$ -partite genuine EPR steering. A tripartite CV GHZ state is a simultaneous eigenstate of  $x_i - x_j$  and  $p_1 + p_2 + p_3$  with eigenvalues 0, and can be prepared as shown in Fig. 2a [11]. For such a state, the following conditional variances are zero ( $g_{x,p}$  are optimised constants) [36]:

$$\begin{aligned} \Delta_{\text{inf}}^2 x_1 &= \Delta^2(x_1 - g_x x_2) = \Delta^2(x_1 - g_x x_3) = 0 \\ \Delta_{\text{inf}}^2 p_1 &= \Delta^2(p_1 - g_p(p_2 + p_3)) = 0 \end{aligned} \quad (4)$$

We see (3) is satisfied and hence, from the symmetry of the eigenstate, the CV GHZ state shows genuine tripartite EPR steering. Extension to higher  $N$  follows.

*Entanglement versus EPR steering:* We now examine whether Results 2 and 3 hold for genuine  $N$ -partite entangled states. Consider the CV Gaussian case. The CV GHZ state shows collective steering, evidenced by the exact predictions for  $x_1$  and  $p_1$  by systems  $\{2, 3\}$ , so that  $\Delta_{\text{inf}} x_1 \Delta_{\text{inf}} p_1 < 1$ . If loss is introduced into beams 2 and 3, we are able to show that collective steering vanishes for efficiencies  $\eta \leq 0.5$ . When  $\eta = 0.5$ , the two symmetric sets of beams  $\{1, 4, 5\}$  and  $\{1, 2', 3'\}$  have the same value for  $\Delta_{\text{inf}} x_1 \Delta_{\text{inf}} p_1$ , and hence by Result 3 cannot satisfy the EPR steering criterion (3). Yet, both sets  $\{1, 2', 3'\}$  and  $\{1, 4, 5\}$  remain genuine tripartite entangled for  $\eta \leq 0.5$  (Fig. 2b), as measurable using two-observable entanglement inequalities [7], and as confirmed by experiment for  $N = 2$  [37]. Similar results

hold for the qubit case. Thus, we conclude genuine entanglement does not imply collective steering, and Result 3 cannot be extended to entanglement inequalities.

*Conclusion:* We have introduced the genuine EPR steering nonlocality, established its potential importance as a resource for secure quantum communications, and derived criteria that can be applied to current experiments known to produce genuine multipartite entanglement. The observation of multipartite EPR steering in any of these systems would seem very feasible.

We thank A. Sambrowski, P. Drummond and S. Armstrong for stimulating discussions. The work was supported by Australian ARC ACQAO COE and DECRA grants. Q. H. thanks support from China NNSF Grant No. 11121091.

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- [32] Otherwise, for some mixture  $P_s$ , we can write  $\langle \prod_{j=1}^N X_j \rangle = \sum_s P_s \langle \prod_{j=1}^N X_j \rangle_s$  where  $\langle \prod_{j=1}^N X_j \rangle_s = \int_{\lambda} P_s(\lambda) \langle \prod_{j \in A_s} X_j \rangle_{s,\lambda} \langle \prod_{j \in B_s} X_j \rangle_{s,\lambda}$ , which gives a contradiction, since at least one of  $P_s$  must be nonzero.
- [33]  $A$  performs measurements to predict  $X_B$  ( $P_B$ ) with uncertainty  $\Delta_{inf,A}X$  ( $\Delta_{inf,A}P$ ) while simultaneously  $C$  performs measurements to predict  $P_B$  ( $X_B$ ) with uncertainty  $\Delta_{inf,C}P$  ( $\Delta_{inf,A}X$ ). The uncertainty principle guarantees  $\Delta_{inf,A}X \Delta_{inf,C}P \geq 1$  and  $\Delta_{inf,C}X \Delta_{inf,A}P \geq 1$ . The result follows.
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